Name: _____

1 True/False

Answer whether the following statements are true or false and briefly explain your answer.

a) [TRUE/FALSE] If A is Turing-recognizable and $\overline{A}$ is Turing-recognizable, then $\overline{A}$ is Turing-decidable. [5 pts]

b) [TRUE/FALSE] Let A be an NP-Hard problem. A poly-time solution to A means that *all* NP-Hard problems are solvable in polynomial time. [5 pts]

c) [TRUE/FALSE] A decision problem A is NP-Hard if and only if SAT $\leq_p A$. [5 pts]

d) [TRUE/FALSE] There exists a pushdown automaton to decide every context-free language. [5 pts]

2 Proofs

a) Let the operator $\diamond$ be defined as follows:

$$A \diamond B = \{st \mid s \in A \text{ and } t \in B \text{ and } |s| = |t|\}$$

Show that Turing-recognizable languages are closed under the $\diamond$ operator. [10 pts]

b) Let $A = \{\langle D \rangle \mid D$ is a DFA that doesn't accept any string containing an odd number of 1s$\}$. Show that A is decidable. [10 pts]

c) Let $\textsc{Comp}_{\text{TM}} = \{\langle M_1, M_2\rangle \mid L(M_1) = \overline{L(M_2)}\}$. Show that $\textsc{Comp}_{TM}$ is undecidable. [10 pts]

d) Let DoubleSat = $\{\langle\Phi\rangle \mid \Phi$ is a Boolean formula with (at least) two different satisfying assignments$\}$. Show that DoubleSat $\in$ NP. [10 pts]